Search in weighted complex networks

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(Received 5 August 2005; published 30 December 2005)

We study trade-offs presented by local search algorithms in complex networks which are heterogeneous in edge weights and node degree. We show that search based on a network measure, local betweenness centrality (LBC), utilizes the heterogeneity of both node degrees and edge weights to perform the best in scale-free weighted networks. The search based on LBC is universal and performs well in a large class of complex networks.

DOI: 10.1103/PhysRevE.72.066128 PACS number(s): 89.75.Fb, 89.75.Hc, 02.10.Ox, 89.70.+c

I. INTRODUCTION

Many large-scale distributed systems found in communications, biology or sociology can be represented by complex networks. The macroscopic properties of these networks have been studied intensively by the scientific community, which has led to many significant results [1–3]. Graph properties such as the degree distribution and clustering coefficient were found to be significantly different from random graphs [4,5] which are traditionally used to model these networks. One of the major findings is the presence of heterogeneity in various properties of the elements in the network. For instance, a large number of the real-world networks including the World Wide Web, the Internet, metabolic networks, phone call graphs, and movie actor collaboration networks are found to be highly heterogeneous in node degree (i.e., the number of edges per node) [1–3]. The clustering coefficients, quantifying local order and cohesiveness [6], were also found to be heterogeneous, i.e., $C(k) \sim k^{-1}$ [7]. These discoveries along with others related to the mixing patterns of complex networks initiated a revival of network modeling in the past few years [1–3]. Focus has been on understanding the mechanisms which lead to heterogeneity in node degree and implications of it on the network properties. It was also shown that this heterogeneity has a huge impact on the network properties and processes such as network resilience [8], network navigation, local search [9], and epidemiological processes [10].

Recently, there have been many studies [11–17] that tried to analyze and characterize weighted complex networks where edges are characterized by capacities or strengths instead of a binary state (present or absent). These studies have shown that heterogeneity is prevalent in the capacity and strength of the interconnections in the network as well. Many researchers [11,13–16] have pointed out that the diversity of the interaction strengths is critical in most real-world networks. For instance, sociologists have shown that the weak links that people have outside their close circle of friends play a key role in keeping the social system together [11]. The Internet traffic [16] or the number of passengers in the airline network [15] are critical dynamical quantities that can be represented by using weighted edges. Similarly, the diversity of the predator-prey interactions and of metabolic reactions is considered as a crucial component of ecosystems [13] and metabolic networks, respectively [14]. Thus it is incomplete to represent real-world systems with equal interaction strengths between different pairs of nodes.

In this paper, we concentrate on finding efficient decentralized search strategies on networks which have heterogeneity in edge weights. This is an intriguing and relatively little studied problem that has many practical applications. Suppose some required information such as computer files or sensor data is stored at the nodes of a distributed network. Then to quickly determine the location of particular information, one should have efficient decentralized search strategies. This problem has become more important and relevant due to the advances in technology that led to many distributed systems such as sensor networks [18], peer-to-peer networks [19] and dynamic supply chains [20]. Previous research on local search algorithms [9,21–24] has assumed that all the edges in the network are equivalent. In this paper we study the complex tradeoffs presented by efficient local search in weighted complex networks. We simulate and analyze different search strategies on Erdős-Rényi (ER) random graphs and scale-free networks. We define a new local parameter called local betweenness centrality (LBC) and propose a search strategy based on this parameter. We show that irrespective of the edge weight distribution this search strategy performs the best in networks with a power-law degree distribution (i.e., scale-free networks). Finally, we show that the search strategy based on LBC is usually equivalent with high-degree search (discussed by Adamic *et al.* [9]) in unweighted (binary) networks. This implies that the search based on LBC is more universal and is optimal in a larger class of complex networks.

The rest of the paper is organized as follows. In Sec. II, we describe the problem in detail and briefly discuss the literature related to search in complex networks. In Sec. III, we define the local betweenness centrality (LBC) of a node's neighbor and show that it depends on the weight of the edge connecting the node and neighbor and on the degree of the neighbor. Section IV explains our methodology and different search strategies considered. Section V gives the details of the simulations conducted for comparing these strategies. In Sec. VI, we discuss the findings from simulations on ER random and scale-free networks. In Sec. VII, we prove that the LBC and degree-based search are equivalent in un-

1539-3755/2005/72(6)/066128(8)/$23.00

weighted networks. Finally, we give conclusions in Sec. VIII.

II. PROBLEM DESCRIPTION AND LITERATURE

The problem of decentralized search goes back to the famous experiment by Milgram [25] illustrating the short distances in social networks. One of the striking observations of this study as pointed out by Kleinberg [21] was the ability of the nodes in the network to find short paths by using only local information. Currently, Watts *et al.* [26] are doing an Internet-based study to verify this phenomenon. Kleinberg demonstrated that the emergence of such phenomenon requires special topological features [21]. Considering a family of network models that generalizes the Watts-Strogatz model [6], he showed that only one particular model among this infinite family can support efficient decentralized algorithms. Unfortunately, the model given by Kleinberg is too constrained and represents only a very small subset of complex networks. Watts *et al.* presented another model to explain the phenomena observed by Milgram which is based upon plausible hierarchical social structures [22]. However, in many real-world networks, it may not be possible to divide the nodes into sets of groups in a hierarchy depending on the properties of the nodes as in the Watts *et al.* model.

Recently, Adamic *et al.* [9] showed that in networks with a power-law degree distribution (scale-free networks) high degree seeking search is more efficient than random walk search. In random walk search, the node that has the message passes it to a randomly chosen neighbor. This process continues until it reaches the target node. In high degree search, the node passes the message to the neighbor that has the highest degree among all nodes in the neighborhood, assuming that a more connected neighbor has a higher probability of reaching the target node. The high degree search was found to outperform the random walk search consistently in networks having power-law degree distribution for different exponents varying from 2.0 to 3.0. Using generating function formalism given by Newman [27], Adamic *et al.* showed that for random walk search the number of steps s until approximately the whole graph is revealed is given by $s \sim N^{3(1-2/\tau)}$, where τ is the power-law exponent, while high degree search leads to a much more favorable scaling $s \sim N^{2-4/\tau}$.

The assumption of equal edge weights (meaning the cost, bandwidth, distance, or power consumption associated with the process described by the edge) usually does not hold in real-world networks. As pointed out by many researchers [11–17], it is incomplete to assume that all the links are equivalent while studying the dynamics of large-scale networks. The total path length (p) in a weighted network for the path 1-2-3$\cdots$-n, is given by $p=\sum_{i=1}^{n} w_{i,i+1}$, where $w_{i,i+1}$ is the weight on the edge from node i to node $i+1$. Even though high-degree search results in a path with smaller number of hops, the total path length may be high if the weights on these edges are high. Thus, to be more realistic and closer to real-world networks we need to explicitly incorporate weights in any proposed search algorithm. In this paper, we are interested in designing decentralized search strategies for networks that have the following properties:

(1) Its node degree distribution follows a power law with exponent varying from 2.0 to 3.0. Although we discuss the search strategies for networks with Poisson degree distribution (ER random graphs), we concentrate more on scale free networks since most of the real world networks are found to exhibit this behavior [1–3].

(2) It has nonuniformly distributed weights on the edges. Here the weights signify the cost or time taken to pass the message or query. Hence, smaller weights correspond to shorter and/or better paths. We consider different distributions such as Beta, uniform, exponential, and power law.

(3) It is unstructured and decentralized. That is, each node has information only about its first and second neighbors and no global information about the target is available. Also, the nodes can communicate only with their immediate neighbors.

(4) Its topology is dynamic (ad hoc) while still maintaining its statistical properties. These particular types of networks are becoming more prevalent due to advances made in different areas of engineering especially in sensor networks [18], peer-to-peer networks [19] and dynamic supply chains [20]. Here, in this paper we analyze the problem of finding decentralized algorithms in such weighted complex networks, which we believe has not been explored to date.

Among the search strategies employed in this paper is a strategy based on the local betweenness centrality (LBC) of nodes. Betweenness centrality (also called load), first developed in the context of social networks [28], has been recently adapted to optimal transport in weighted complex networks by Goh *et al.* [17]. These authors have shown that in the strong disorder limit (that is, when the total path length is dominated by the maximum edge weight over the path), the load distribution follows a power law for both ER random graphs and scale-free networks. To determine a node's betweenness as defined by Goh *et al.* one would need to have the knowledge of the entire network. Here we define a local parameter called local betweenness centrality (LBC) which only uses information on the first and second neighbors of a node, and we develop a search strategy based on this local parameter.

III. LOCAL BETWEENNESS CENTRALITY

We assume that each node in the network has information about its first and second neighbors. For calculating the local betweenness centrality of the neighbors of a given node we consider the local network formed by that node (which we will call the root node), its first and second neighbors. Then, the betweenness centrality, defined as the fraction of shortest paths going through a node [3], is calculated for the first neighbors in this local network. Let $L(i)$ be the LBC of a neighbor node i in the local network. Then $L(i)$ is given by

$$L(i) = \sum_{\substack{s \neq i \neq t \\ s,t \in \text{local network}}} \frac{\sigma_{st}(i)}{\sigma_{st}}$$

where σ_{st} is the total number of shortest paths (where shortest path means the path over which the sum of weights is

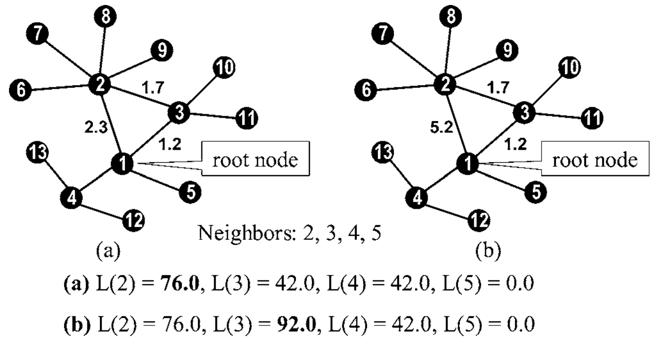

FIG. 1. (a) In this configuration, neighbor node 2 has a higher LBC than other neighbors 3, 4, and 5. This depicts why higher degree for a node helps in obtaining higher LBC. (b) However, in this configuration the LBC of the neighbor node 3 is higher than neighbors 2, 4, and 5. This is due to the fact that the edge connecting 1 and 2 has a larger weight. These two configurations show that the LBC of a neighbor depends both on the edge weight and the node degree. In both cases, edge weights other than those shown in the figure are assumed to be 1.

minimal) from node s to t. $\sigma_{st}(i)$ is the number of these shortest paths passing through i. If the LBC of a node is high, it implies that this node is critical in the local network. Intuitively, we can see that the LBC of a neighbor depends on both its degree and the weight of the edge connecting it to the root node. For example, let us consider the networks in Figs. 1(a) and 1(b). Suppose that these are the local networks of node 1. In the network in Fig. 1(a), node 2 has the highest degree among the neighbors of node 1 (i.e., nodes 2, 3, 4, and 5). All the shortest paths from the neighbors of node 2 (6, 7, 8, and 9) to other nodes must pass through node 2. Hence, we see that higher degree for a node definitely helps in obtaining a higher LBC.

Now consider a similar local network but with a higher weight on the edge from 2 to 1 as shown in Fig. 1(b). In this network all the shortest paths through node 2 will also pass through node 3 (2-3-1) instead of going directly from node 2 to node 1. Hence, the LBC of the neighbor node 3 will be higher than that of neighbor 2. Thus we clearly see that the LBCs of the neighbors of node 1 depend on both the neighbors' degrees and the weights on the edges connecting them. Note that a neighbor having the highest degree or the smallest weight on the edge connecting it to root node does not necessarily imply that it will have the highest LBC.

IV. METHODOLOGY

In unweighted scale-free networks, Adamic *et al.* [9] have shown that high degree search which utilizes the heterogeneity in node degree is efficient. Thus one expects that in weighted power-law networks, an efficient search strategy should consider both the edge weights and node degree. We investigated the following set of search strategies given in the order of the amount of information required.

(1) *Choose a neighbor randomly*: The node tries to reach the target by passing the message/query to a randomly selected neighbor.

(2) *Choose the neighbor with smallest edge weight*: The node passes the message along the edge with minimum weight. The idea behind this strategy is that by choosing a neighbor with minimum edge weight the expected distance traveled would be less.

(3) *Choose the best-connected neighbor*: The node passes the message to the neighbor which has the highest degree. The idea here is that by choosing a neighbor which is well-connected, there is a higher probability of reaching the target node. Note that this strategy takes the least number of hops to reach the target [9].

(4) *Choose the neighbor with the smallest average weight*: The node passes the message to the neighbor which has the smallest average weight. The average weight of a node is the average weight of all the edges incident on that node. The idea here is similar to the second strategy. Instead of passing the message greedily along the least weighted edge, the algorithm passes to the node that has the minimum average weight.

(5) *Choose the neighbor with the highest LBC*: The node passes the message to the neighbor which has the highest LBC. A neighbor with highest LBC would imply that many shortest paths in the local network pass through this neighbor and the node is critical in the local network. Thus, by passing the message to this neighbor, the probability of reaching the target node quicker is higher.

Note that the strategy which depends on LBC utilizes slightly move information than strategy 4, namely the edge weights between second neighbors, but it is considerably more informative, it reflects the heterogeneities in both edge weights and node degree. Thus we expect that this search will perform better than the others, that is, it will give smaller path lengths than the others.

V. SIMULATIONS

For comparing the search strategies we used simulations on random networks with Poisson and power-law degree distributions. For homogeneous networks we used the Poisson random network model given by Erdős and Rényi [4]. We considered a network on N nodes where two nodes are connected with a connection probability p. For scale-free networks, we considered different values of degree exponent τ ranging from 2.0 to 3.0 and a degree range of $2 < k < m \sim N^{1/\tau}$ and generated the network using the method given by Newman [27]. Once the network was generated, we extracted the largest connected component, shown to always exist for $2 < \tau < 3.48$ [29] and in ER networks for $p > 1/N$ [5]. We did our analysis on this largest connected component that contains the majority of the nodes after verifying that the degree distribution of this largest connected component is nearly the same as in the original graph. The weights on the edges were generated from different distributions such as Beta, uniform, exponential and power law. We considered these distributions in the increasing order of their variances to understand how the heterogeneity in edge weights affects different search strategies.

Further, we randomly choose K pairs (source and target) of nodes. The source, and consecutively each node receiving

TABLE I. Comparison of search strategies in a Poisson random network. The edge weights were generated randomly from an exponential distribution with mean 5 and variance 25. The values in the table are the average path distances obtained for each search strategy in these networks. The strategy which passes the message to the neighbor with the least edge weight performs the best.

Search strategy	500 nodes	1000 nodes	1500 nodes	2000 nodes
Random walk	1256.3	2507.4	3814.9	5069.5
Minimum edge weight	**597.6**	**1155.7**	**1815.5**	**2411.2**
Highest degree	979.7	1923.0	2989.2	3996.2
Minimum average node weight	832.1	1652.7	2540.5	3368.6
Highest LBC	864.7	1800.7	2825.3	3820.9

the message, sends the message to one of its neighbors depending on the search strategy. The search continues until the message reaches the node whose neighbor is the target node. In order to avoid passing the message to a neighbor that has already received it, a list l_i of all the neighbors that received the message is maintained at each node i. During the search process, if node i passes the message to its neighbor j, which does not have any more neighbors that are not in the list l_j, then the message is routed back to the node i. This particular neighbor j is marked to note that this node cannot pass the message any further. The average path distance was calculated for each search strategy from the paths obtained for these K pairs. We repeated this simulation for 10 to 50 instances of the Poisson and power-law networks depending on the size of the network.

VI. ANALYSIS

First, we study and compare different search strategies on ER random graphs. The weights on the edges were generated from an exponential distribution with mean 5 and variance 25. Table I compares the performance of each strategy for the networks of size 500, 1000, 1500, and 2000 nodes. We took the connection probability to be $p=0.004$ and hence a giant connected component always exists [5]. From Table I, it is evident that the strategy which passes the message to the neighbor with the least edge weight is better than all the other strategies in homogeneous networks. Remarkably, a search strategy that needs less information than other strategies (3, 4, and 5), performed best, while high degree search and LBC did not perform well since the network is highly homogenous in node degree.

However, if we decrease the heterogeneity in edge weights (use a distribution with lesser variance), we observe that high LBC search performs best (see Table II). In conclusion, when the heterogeneity of edge weights is high compared to the relative homogeneity of node degrees, the search strategies which are purely based on edge weights would perform better. However, as the heterogeneity of the edge weights decrease the importance of edge weights decreases and strategies which consider both edge weights and node degree perform better.

Next we investigated how the search strategies perform on scale-free networks. Figure 2 shows the scaling of different search strategies for scale-free networks with exponent 2.1. As conjectured, the search strategy that utilizes the heterogeneities of both the edge weights and nodes' degrees (the high LBC search) performed better than the other strategies. A similar phenomenon was observed for different exponents of the scale-free network (see Table III). Except for the power-law exponent 2.9, the high LBC search was consistently better than others. We observe that as the heterogeneity in the node degree decreases (i.e., as power-law exponent increases), the difference between the high LBC search and other strategies decreases. When the exponent is 2.9, the performance of LBC, minimum edge weight and high degree searches were almost the same. Note that when the network becomes homogeneous in node degree the minimum edge weight search performs better than high LBC search (Table

TABLE II. Comparison of search strategies in a Poisson random network with 2000 nodes. The table gives results for different edge weight distributions. The mean for all the distributions is 5 and variance is σ^2. The values in the table are the average path lengths obtained for each search strategy in these networks. When the weight heterogeneity is high, the minimum edge weight search strategy was the best. However, when the heterogeneity of edge weights is low, then LBC performs better.

Search strategy	Beta $\sigma^2=2.3$	Uniform $\sigma^2=8.3$	Exp. $\sigma^2=25$	Power law $\sigma^2=4653.8$
Random walk	1271.91	1284.9	1253.68	1479.32
Minimum edge weight	1017.74	767.405	577.83	562.39
Highest degree	994.64	1014.05	961.5	1182.18
Minimum average node weight	1124.48	954.295	826.325	732.93
Highest LBC	980.65	968.775	900.365	908.48

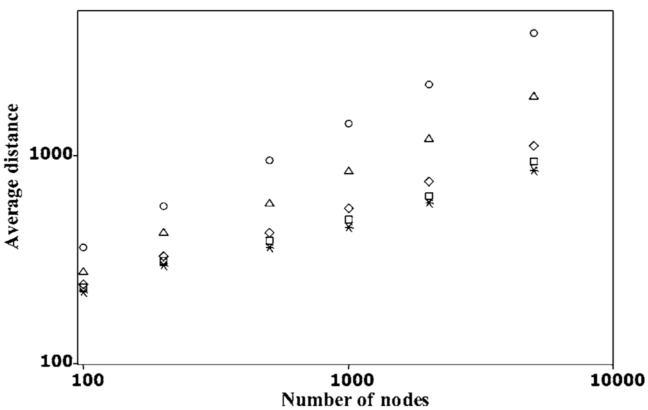

FIG. 2. Scaling for search strategies in power-law networks with exponent 2.1. The edge weights are generated from an exponential distribution with mean 10 and variance 100. The symbols represent random walk (○) and search algorithms based on minimum edge weight (□), high degree (◇), minimum average node weight (△), and high LBC (*).

I). This implies that similarly to high degree search [9], the effectiveness of high LBC search also depends on the heterogeneity in node degree.

Table IV shows the performance of all the strategies on a scale-free network (exponent 2.1) with different edge weight distributions. The percentage values in the brackets show by how much the average distance for that search is higher than the average distance obtained by the high LBC search. As in random graphs, we observe that the impact of edge weights on search strategies increases as the heterogeneity of the edge weights increase. For instance, when the variance (heterogeneity) of edge weights is small, high degree search is better than the minimum edge weight search. On the other hand, when the variance (heterogeneity) of edge weights is high, the minimum edge weight strategy is better than high degree search. In each case, the high LBC search which reflects both edge weights and node degree always outperformed the other strategies. Thus, it is clear that in power-law networks, irrespective of the edge weight distribution and the power-law exponent, high LBC search always performs better than the other strategies (Tables III and IV).

Figure 3 gives a pictorial comparison of the behavior of

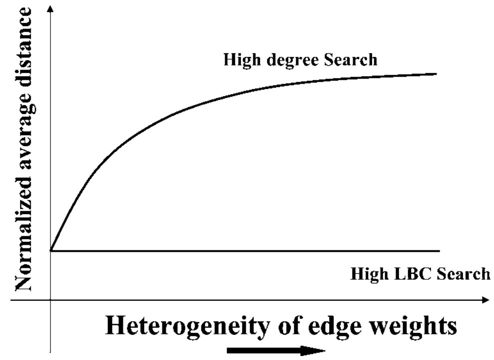

FIG. 3. The pictorial comparison of the behavior of high degree and high LBC search as the heterogeneity of edge weights increases in power-law networks. Note that average distances are normalized with respect to high LBC search.

high degree and high LBC search as the heterogeneity of the edge weights increase (based on the results shown in Table IV). Since many studies [11–17] have shown that there is a large heterogeneity in the capacity and strengths of the interconnections in the real networks, it is important that local search is based on LBC rather than high degree as shown by Adamic *et al.* [9].

Note that LBC has been adopted from the definition of betweenness centrality (BC) which requires the global knowledge of the network. BC is defined as the fraction of shortest paths among all nodes in the network that pass through a given node and measures how critical the node is for optimal transport in complex networks. In unweighted scale-free networks there exists a scaling relation between node betweenness centrality and degree, $BC \sim k^{\eta}$ [30]. This implies that the higher the degree, the higher is the BC of the node. This may be the reason why high degree search is optimal in unweighted scale-free networks (as shown by Adamic *et al.* [9]). However, Goh *et al.* [17] have shown that no scaling relation exists between node degree and betweenness centrality in weighted complex networks. It will be interesting to see the relationship between local and global betweenness centrality in our future work. Also, note that the minimum average node weight strategy (strategy 4) uses only slightly less information than LBC search. However, LBC search consistently and significantly outperforms it (see

TABLE III. Comparison of search strategies in power-law network on 2000 nodes with different power-law exponents. The edge weights are generated from an exponential distribution with mean 5 and variance 25. The values in the table are the average path lengths obtained for each search strategy in these networks. LBC search, which reflects both the heterogeneities in edge weights and node degree, performed the best for all power-law exponents. The systematic increase in all path lengths with the increase of the power-law exponent τ is due to the fact that the average degree of the network decreases with τ.

Power-law exponent= Search strategy	**2.1**	**2.3**	**2.5**	**2.7**	**2.9**
Random walk	1108.70	1760.58	2713.11	3894.91	4769.75
Minimum edge weight	318.95	745.41	1539.23	2732.01	3789.56
Highest degree	375.83	761.45	1519.74	2693.62	3739.61
Minimum average node weight	605.41	1065.34	1870.43	3042.27	3936.03
Highest LBC	**298.06**	**707.25**	**1490.48**	**2667.74**	**3751.53**

TABLE IV. Comparison of search strategies in power-law networks with exponent 2.1 and 2000 nodes with different edge weight distributions. The mean for all the edge weight distributions is 5 and the variance is σ^2. The values in the table are the average distances obtained for each search strategy in these networks. The values in the brackets show the relative difference between average distance for each strategy with respect to the average distance obtained by the LBC strategy. LBC search, which reflects both the heterogeneities in edge weights and node degree, performed the best for all edge weight distributions.

Search strategy	Beta $\sigma^2=2.3$	Uniform $\sigma^2=8.3$	Exp. $\sigma^2=25$	Power law $\sigma^2=4653.8$
Random walk	1107.71 (202%)	1097.72 (241%)	1108.70 (272%)	1011.21 (344%)
Minimum edge weight	704.47 (92%)	414.71 (29%)	318.95 (7%)	358.54 (44%)
Highest degree	379.98 (4%)	368.43 (14%)	375.83 (26%)	394.99 (59%)
Minimum average node weight	1228.68 (235%)	788.15 (145%)	605.41 (103%)	466.18 (88%)
Highest LBC	**366.26**	**322.30**	**298.06**	**247.77**

Tables I–IV). This implies that LBC search uses the information correctly.

VII. LBC ON UNWEIGHTED NETWORKS

In this section, we show that the neighbor with the highest LBC is usually the same as the neighbor with the highest degree in unweighted networks. Hence, high LBC search would give identical results as high degree search in unweighted networks. As mentioned earlier, in unweighted scale-free networks, there is a scaling relation between the (global) BC of a node and its degree, as $BC \sim k^{\eta}$ [30]. However, this does not imply that in an unweighted local network the neighbor with highest LBC is always the same as the neighbor with the highest degree. Here, we show that in most cases the highest degree and the highest LBC neighbors coincide. First, let us consider a tree-like local network without any loops similar to the network configuration shown in Fig. 4(a). In a local network, there are three types of nodes, namely, root node, first neighbors and second neighbors. Let the degree of the root node be d and the degree of the neighbors be $k_1, k_2, k_3, \ldots, k_d$. The number of nodes ($n$) in the local network is $n = 1 + \Sigma_{j=1}^{d} k_j$ [one root node, d first neighbors and $\Sigma_{j=1}^{d}(k_j - 1)$ second neighbors]. In a tree network there is a single shortest path between any pair of nodes s and t, thus $\sigma_{st}(i)$ is either zero or one. Then the LBC of a first neighbor i is given by $L(i) = (k_i - 1)(n-2) + (k_i - 1)(n - k_i)$ where k_i is the degree of the neighbor. The first term is due to the shortest paths from $k_i - 1$ neighbors of node i to $n-2$ remaining nodes (other than node i and the neighbor j) in the network. The second term is due to the shortest paths from $n - k_i$ nodes (other than $k_i - 1$ neighbors and node i) to $k_i - 1$ neighbors of node i. Note that we choose not to explicitly take into account the symmetry of distance in undirected networks and count the s-t and t-s paths separately. $L(i)$ is an increasing function if $k_i < n - \frac{1}{2}$, a condition that is always satisfied since $n = 1 + \Sigma_{j=1}^{d} k_j$. This implies that in a local network with treelike structure, the neighbor with highest degree has the highest LBC. We extend the above result for other configurations of the local network by considering different possible cases.

The possible edges other than the edges present in a tree-like local network are an edge between two first neighbors, an edge between a first neighbor and a second neighbor and an edge between two second neighbors. As shown in Fig. 4(b), an edge among two first neighbors changes the LBC of the root node but not that of the neighbors. Figure 4(c) shows a configuration of a local network with an edge added between a first and a second neighbor. Now, there is a small change in the LBCs of the neighbors (nodes 2 and 3) which are connected to a common second neighbor (node 9). Since

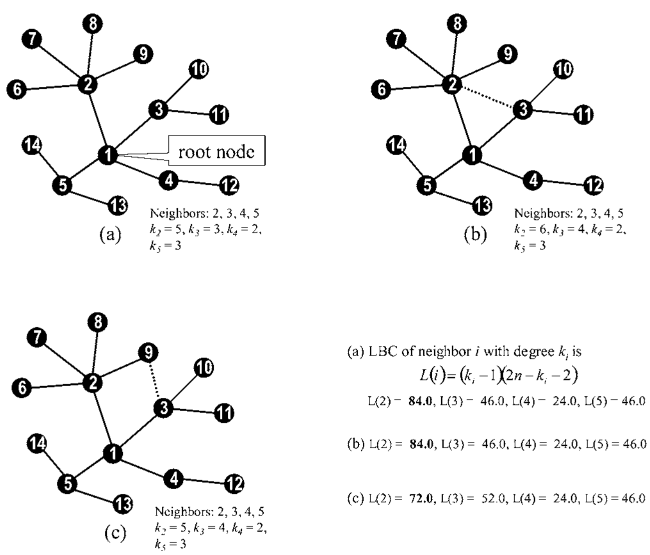

FIG. 4. (a) A configuration of a local network with a tree like structure. In such local networks, the neighbor with the highest degree has the highest LBC. (b) A local network with an edge between two first neighbors. Here again the neighbor with the highest degree has the highest LBC. (c) A local network with an edge between a first neighbor and a second neighbor. Although there is change in LBCs of neighbors, the order remains the same.

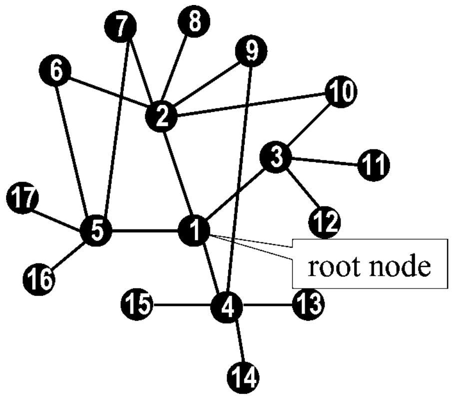

FIG. 5. An instance of a local network where the order of neighbors with respect to LBC is not the same as the order with respect to node degree.

node 9 is now shared by neighbors 2 and 3, the LBC contributed by node 9 is divided between these two neighbors. The LBC of such a neighbor i is $L(i)=(k_i-2)(n-2)+(k_i-2)(n-k_i)+(n-k_j-1)$ where k_i is the degree of the neighbor i and k_j is the degree of the neighbor with which node i has a common second neighbor. The decrease in the LBC of neighbor i is $(n-k_i+k_j-1)$. If there are two neighbors with the same degree (one with a common second neighbor and another without any) then the neighbor without any common second neighbors will have higher LBC. Another possible change of order with respect to LBC would be with a neighbor l of degree $k_l=k_i-1$ (if it exists). However, $L(i)-L(l)=(n-k_i-k_j+1)$ is always greater than 0, since $n=\sum_{j=1}^{d}k_j$ in this local network. Thus the only scenario under which the order of neighbors with respect to LBC is different than their order with respect to degree when adding an edge between first and second neighbors is if that creates two first neighbors with the same degree. A similar argument leads to an identical conclusion in the case of adding an edge between two second neighbors as well.

The above discussion suggests that the highest degree neighbor is always the same as the highest LBC neighbor. This is not true in few peculiar instances of local networks. For example, consider the network shown in Fig. 5 which has several edges between the first and second neighbors. We see that the highest degree neighbor is not the same as the highest LBC neighbor. In this local network, the highest degree first neighbor (node 2), participates in several four-node circuits that include the root node. Thus, there are multiple shortest paths starting from second-neighbor nodes on these cycles (nodes 6, 7, 9, 10) and the contributions to node 2's LBC from the paths that pass through it are smaller than unity, consequently the LBC of node 2 will be relatively small. This may be one of the reasons why the highest-degree neighbor node 2 is not the highest LBC neighbor. We feel that this happens only in some special instances of local networks. From about 50 000 simulations we found that in 99.63% of cases the highest degree neighbor is the same as the highest LBC neighbor. Hence, we can conclude that in unweighted networks the neighbor with highest LBC is usually identical to the neighbor with the highest degree.

VIII. CONCLUSION

In this paper we have given a new direction for local search in complex networks with heterogeneous edge weights. We proposed a local search algorithm based on a new local measure called local betweenness centrality. We studied complex tradeoffs presented by efficient local search in weighted complex networks and showed that heterogeneity in edge weights has huge impact on search. Moreover, the impact of edge weights on search strategies increases as the heterogeneity of the edge weights increase. We also demonstrated that the search strategy based on LBC utilizes the heterogeneity in both the node degree and edge weight to perform the best in power-law weighted networks. Furthermore, we have shown that in unweighted power-law networks the neighbor with the highest degree is usually the same as the neighbor with the highest LBC. Hence, our proposed search strategy based on LBC is more universal and is efficient in a larger class of complex networks.

ACKNOWLEDGMENTS

The authors would like to acknowledge the National Science Foundation (Grant No. SST 0427840) and a Sloan Research Fellowship to one of the authors (R. A.) for making this work feasible. Any opinions, findings and conclusions or recommendations expressed in this material are those of the author(s) and do not necessarily reflect the views of the National Science Foundation (NSF). In addition, the first author (H.P.T.) would like to thank Usha Nandini Raghavan for interesting discussions on issues related to this work.

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